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SPECIAL PRODUCTS AND FACTORIZATION

- **Special Products:** Products like 108×108, 97×97, 104×96 can easily be calculated with the help of $(a + b)^2$, $(a - b)^2$, $(a + b)(a - b)$ respectively. Such products are called special products.

Special Product Formula :

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$(a + b)^2 - (a - b)^2 = 4ab$

$(a + b)(a - b) = a^2 - b^2$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x - a)(x - b) = x^2 - (a + b)x + ab$

$(a - b)^3 = a^3 - 3ab(a - b) - b^3$

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$

$(a + b)(a^2 - ab + b^2) = a^3 + b^3$

$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

- **Factorization of polynomials:** Factorization of polynomials is a process of writing the polynomial as a product of two (or more) polynomials. Each polynomial in the product is called a factor of the given polynomial.
- **Method of factorization:** Factorization by distributive property.

  Factorization involving the difference of two squares.

  Factorizaion of a perfect square polynomial.

  Factorization of a polynomial reducible to the difference of two squares.

  Factorization of perfect cube polynomials.

  Factorization of polynomials involving sum or difference of two cubes.

  Factorizing trinomials by splitting the middle term.
- **HCF of polynomials:** HCF of two or more given polynomials is the product of the polynomials of highest degree and greatest numerical coefficient each of which is a factor of each of the given polynomials.
- **LCM of polynomials:** LCM of two or more polynomials is the product of the polynomials of the lowest degree and the smallest numerical coefficient which are multiples of the corresponding elements of each of the given polynomials.
- **Rational Expression:** An algebraic expression which can be expressed in the form $\frac{p}{q}$ where p is any polynomial and q is non-zero polynomial. A rational expression need not to be a polynomial. Every polynomial is a rational expression also.
- **Operations on rational expressions:** Four fundamental operations $(+, -, \times, \div)$ on rational expressions are performed in exactly the same way as in the case of rational numbers.

  Result of multiplication of rational expressions must be in the lowest terms or in lowest form.

  Sum, difference, product and quotient of two rational expressions are also rational expressions.
- **Reciprocal expression:** $\frac{S}{R}$ is the reciprocal expression of $\frac{R}{S}$.

  We use reciprocal expression in division of two rational expressions as $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R}$.
- **Conversion of a rational expression into lowest terms:** Cancel the common factor if any from the numerator and denominator of the rational expression.

CHECK YOUR PROGRESS:

1. Which of the following is not a rational expression?

   (A) $\sqrt{21}$ (B) $x+\frac{1}{x}$ (C) $8\sqrt{x}+6\sqrt{y}$ (D) $\frac{x+\sqrt{2}}{x-\sqrt{2}}$

2. $\left(a^2+b^2\right)^2+\left(a^2-b^2\right)^2$ equals to :

   (A) $2\left(a^2+b^2\right)$ (B) $4\left(a^2+b^2\right)$ (C) $4\left(a^4+b^4\right)$ (D) $2\left(a^4+b^4\right)$

3. If $m-\frac{1}{m}=-\sqrt{3}$, then $m^3-\frac{1}{m^3}$ equals to :

   (A) $-6\sqrt{3}$ (B) $-3\sqrt{3}$ (C) 0 (D) $6\sqrt{3}$

4. $\frac{327\times327-323\times323}{327+323}$ equals to:

   (A) 650 (B) 327 (C) 323 (D) 4

5. $8m^3-n^3$ equals to:

   (A) $(2m-n)\left(4m^2-2mn+n^2\right)$

   (B) $(2m-n)\left(4m^2+2mn+n^2\right)$

   (C) $(2m-n)\left(4m^2+4mn+n^2\right)$

   (D) $(2m+n)\left(4m^2+2mn+n^2\right)$

6. Find the sum of $\frac{x+2}{x-2}$ and $\frac{x-2}{x+2}$

7. Find the LCM of x^2-1 and x^2-x-2.

8. Find the HCF of $36x^5y^2$ and $90x^3y^4$.

9. Factorise (i) x^4-81y^4 (ii) $5x^2-8x-4$.

10. Simplify the following:

    $$\frac{6x^2+17x+12}{10x^2+17x+3}\div\frac{6x^2-7x-20}{10x^2-23x-5}.$$

STRETCH YOURSELF :

1. If $a^4 + \frac{1}{a^4} = 34$, find $a^3 - \frac{1}{a^3}$.
2. Find the sum of $\frac{x+1}{x-1}$ and its reciprocal.
3. Without actual multiplication evaluate $103 \times 103 \times 103$.
4. Find the value of $x^3 - y^3$ when $x - y = 5$ and $xy = 66$.

ANSWERS

CHECK YOUR PROGRESS:

1. C 2. D 3. A
4. D 5. B

6. $\frac{2x^2+8}{x^2-4}$ 7. $(x^2-1)(x-2)$
8. $18x^3y^2$
9. (i) $(x^2+9y^2)\,(x+3y)\,(x-3y)$

   (ii) $(x-2)\,(5x+2)$
10. 1

STRETCH YOURSELF :

1. 14 2. $\frac{2(x^2+1)}{(x^2-1)}$
3. 1092727 4. 1115